 Can we understand beta decay rates in general? Nov. 15, 2005

first half page of nuclides chart (symbol ε stands for electron capture/ β+ decay)

→ 27 isotopes: 8 β^- decays, 6 β^+ decays, spanning 16 orders of magnitude in rate!

	Z	A	Atomic mass (u)	I^π	*Abundance* or Half-life
H	1	1	1.007825	$\frac{1}{2}^+$	*99.985%*
		2	2.014102	1^+	*0.015%*
		3	3.016049	$\frac{1}{2}^+$	12.3 y (β^-)
He	2	3	3.016029	$\frac{1}{2}^+$	*1.38 × 10⁻⁴%*
		4	4.002603	0^+	*99.99986%*
Li	3	6	6.015121	1^+	*7.5%*
		7	7.016003	$\frac{3}{2}^-$	*92.5%*
		8	8.022486	2^+	0.84 s (β^-)
Be	4	7	7.016928	$\frac{3}{2}^-$	53.3 d (ε)
		8	8.005305	0^+	0.07 fs (α)
		9	9.012182	$\frac{3}{2}^-$	*100%* slowest
		10	10.013534	0^+	$\boxed{1.6\ \text{My}\ (\beta^-)}$
		11	11.021658	$\frac{1}{2}^+$	13.8 s (β^-)
B	5	8	8.024606	2^+	0.77 s (ε)
		9	9.013329	$\frac{3}{2}^-$	0.85 as (α)
		10	10.012937	3^+	*19.8%*
		11	11.009305	$\frac{3}{2}^-$	*80.2%*
		12	12.014353	1^+	20.4 ms (β^-)
		13	13.017780	$\frac{3}{2}^-$	17.4 ms (β^-)
C	6	9	9.031039	$\frac{3}{2}^-$	0.13 s (ε)
		10	10.016856	0^+	19.2 s (ε)
		11	11.011433	$\frac{3}{2}^-$	20.4 m (ε)
		12	12.000000	0^+	*98.89%*
		13	13.003355	$\frac{1}{2}^-$	*1.11%*
		14	14.003242	0^+	5730 y (β^-)
		15	15.010599	$\frac{1}{2}^+$	2.45 s (β^-) fastest
N	7	12	12.018613	1^+	$\boxed{11\ \text{ms}\ (\varepsilon)}$
		13	13.005739	$\frac{1}{2}^-$	9.96 m (ε)
		14	14.003074	1^+	*99.63%*
		15	15.000109	$\frac{1}{2}^-$	*0.366%*
		16	16.006100	2^-	7.13 s (β^-)

Some anomalies:

1. According to our theory, the very slow decay: (1.6 x 10^6 yrs)

$$ {}^{10}_{4}\mathrm{Be}\,(0^+) \rightarrow {}^{10}_{5}\mathrm{B}\,(3^+) + e^- + \overline{\nu}_e $$

should not occur at all, because angular momentum does not add up, i.e.:

$$ \vec{0} \neq \vec{3} + (\vec{0} \;\; or \;\; \vec{1}) $$

2. Another example: (16.1 hr)

$$ {}^{76}_{35}\mathrm{Br}\,(1^-) \rightarrow {}^{76}_{34}\mathrm{Se}\,(0^+) + e^+ + \nu_e $$

This should not occur because the wavefunctions in the nuclear matrix element have opposite parity, so the integrand is odd and should vanish:

$$ M_{nuclear} \equiv \int \psi_f^*(\vec{r})\;\psi_i(\vec{r})\,d^3r = 0 \quad ??? $$

Forbidden Decays:

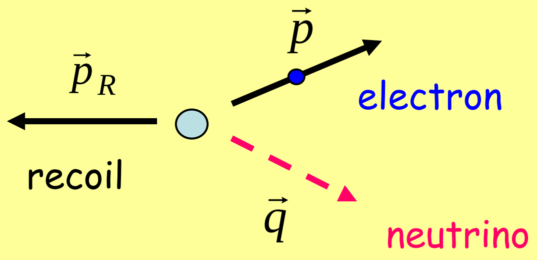
These are two examples of forbidden decays – they cannot proceed under the allowed approximation, since

$$M_{if} = G \int \psi^*_{p,f}(\vec{r})\, \phi^*_e(\vec{r})\, \phi^*_\nu(\vec{r})\, \psi_{n,i}(\vec{r})\; d^3 r = 0 \quad if \quad \phi^*_e(\vec{r})\, \phi^*_\nu(\vec{r}) = \frac{1}{V}$$

Is there some other way they can occur?

Reconsider the electron and antineutrino wave function as a multipole expansion:

$$V\, \phi^*_e(\vec{r})\, \phi^*_\nu(\vec{r}) = e^{i\vec{p}_R \bullet \vec{r}/\hbar} \equiv \sum_{L=0}^{\infty} i^L (2L+1)\; j_L(p_R r/\hbar)\; P_L(\cos\theta)$$



$$j_L = spherical \quad Bessel \quad Function$$

$$P_L(\cos\theta) = Legendre \quad polynomial$$

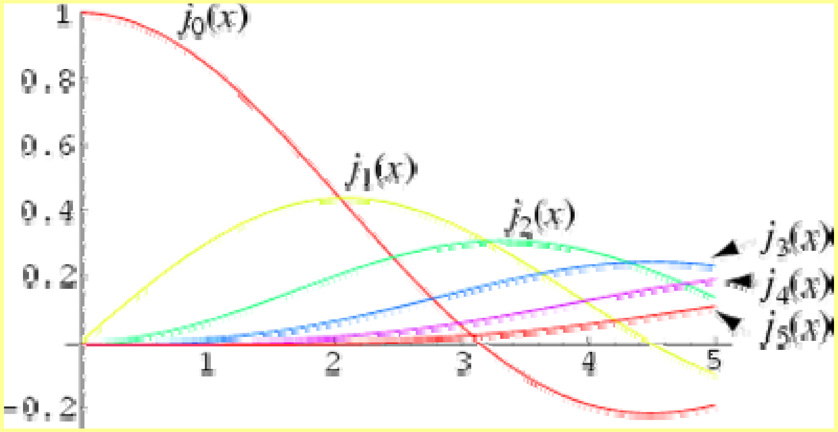
Beyond the "allowed approximation":

$$e^{i\vec{p}_R \bullet \vec{r}/\hbar} \equiv \sum_{L=0}^{\infty} i^L \,(2L+1)\; j_L(p_R r/\hbar)\; P_L\,(\cos\theta)$$

spherical Bessel functions:

$$j_o(x) = \frac{\sin x}{x}; \quad j_1(x) = \frac{\sin x}{x^2} - \frac{\cos x}{x} \ldots.$$

$$with \;\; x = p_R r/\hbar$$

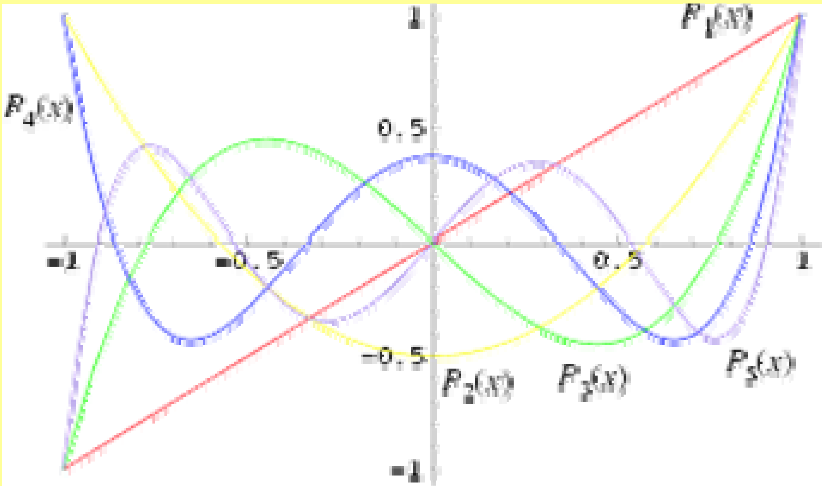


for successively larger L, they become more significant for larger recoil momentum
→ this will change the momentum dependence of our prediction !

Legendre polynomials:

$$P_o = 1, \quad P_1 = x, \quad P_2 = \frac{1}{2}(3x^2 - 1) \ldots.$$

$$with \;\; x = \cos\theta$$



these introduce a new angular dependence to the integrand for M_{if} → equivalent to angular momentum L

Effect on beta transitions:

- angular momentum coupling for the multipole order L, together with S and nuclear angular momentum allows previously impossible reactions to proceed

- multipole term has parity $(-1)^L$, which allows nuclear states of opposite parity to be "connected" by the beta decay operator

- momentum dependence of the matrix element varies as $(p_R r/\hbar)^L$ ...

  since this is small, the lowest multipole order L that satisfies the conservation laws will dominate the transition

  rate ~ $|M|^2 \sim (p_R\, r/\hbar)^{2L} \cong (0.01)^{2L} \rightarrow$ dramatically smaller for large L

  momentum dependence also affects the shape of the spectrum; Kurie plots are not linear unless "shape factors" are taken into account....

- naming convention:

L = 0	allowed
L = 1	first forbidden
L = 2	second forbidden
L = 3	third forbidden....

Classification: all known decays....

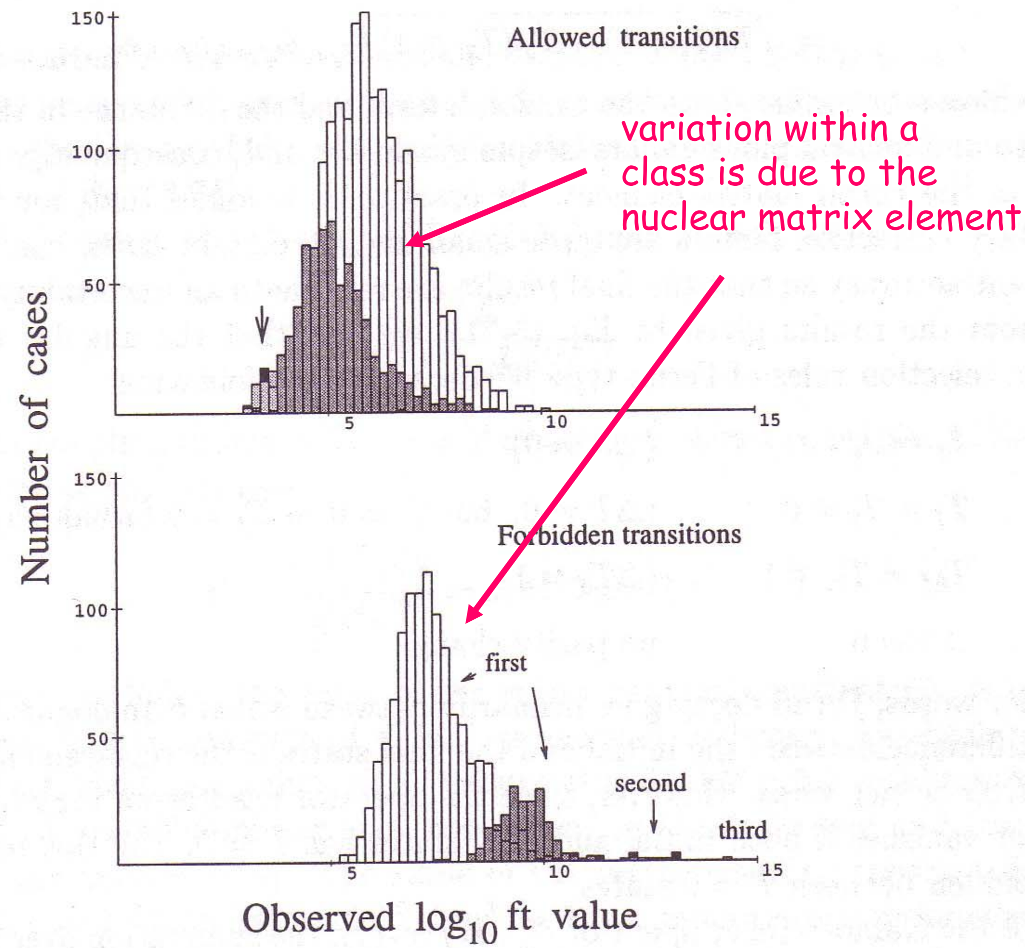
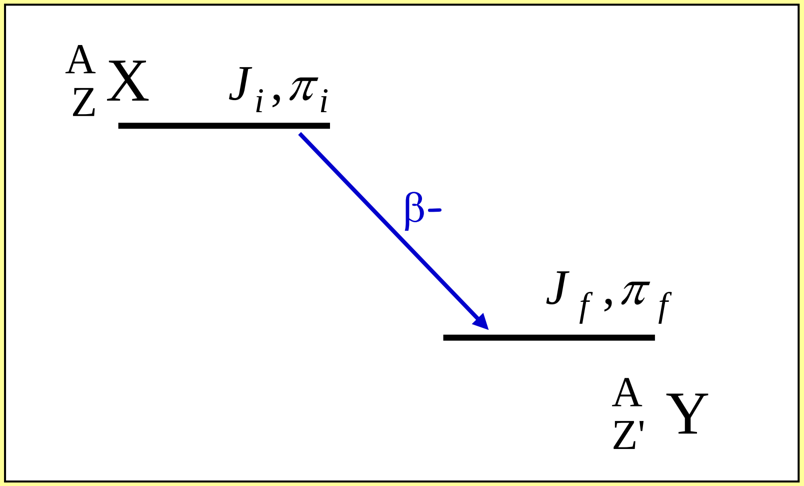
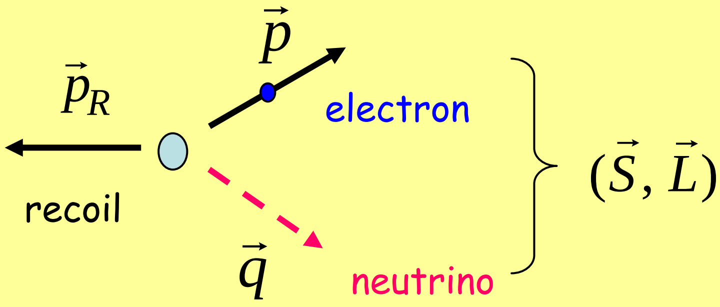


Figure 5-8: Systematics of observed log ft values. The grey area in the upper panel shows 718 cases of $0^+ \rightleftharpoons 1^+$ allowed transitions, and the remaining 1741 cases of other allowed decays are shown by the white histogram. The peak of the distribution for the 24 cases of $0^+ \to 0^+$ superallowed decay is indicated by the arrow.

Classification of decays: general

Nuclear case: $${}^{A}_{Z}X \rightarrow {}^{A}_{Z'}Y + e^{-} + \bar{\nu}_e$$





Conservation laws:

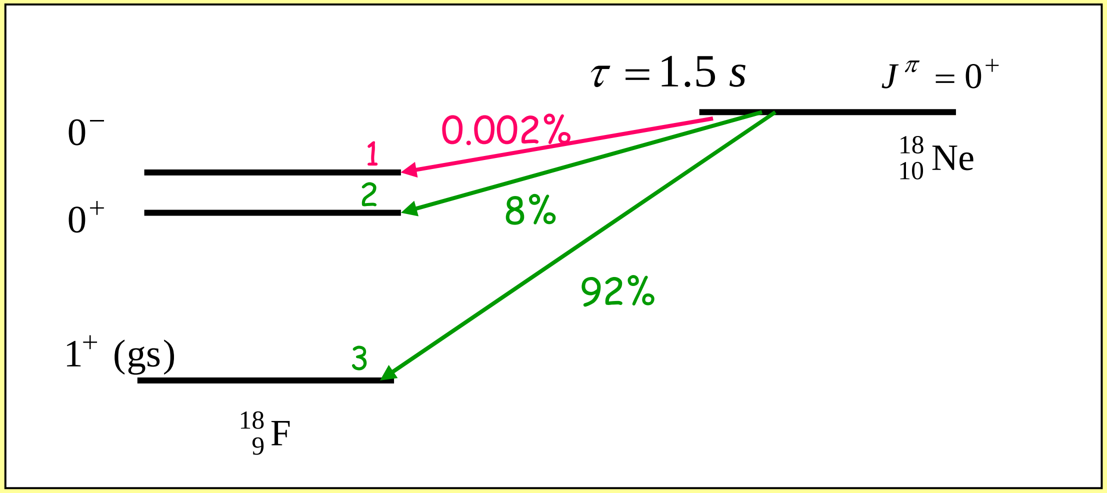
$$\vec{J}_i = \vec{J}_f + \vec{S} + \vec{L}$$

$$\pi_i = \pi_f (-1)^L$$

with S = 0 (Fermi) or S = 1 (Gamow-Teller)

Smallest value of L that is consistent with conservation laws will dominate the transition.

Example: β+ decay of ^{18}Ne



Branching ratio (BR): the fraction of decays that go to a particular final state.

In this case, $\lambda_{total} = 1/\tau = 0.667\ sec^{-1}$; $\lambda = \lambda_1 + \lambda_2 + \lambda_3$, with $\lambda_i = BR(i)\ \lambda_{total}$

Transition 1: **$0^+ \rightarrow 0^-$** This is a **first forbidden GT decay**, with the slowest partial rate:

$$\vec{0} = \vec{0} + \vec{S} + \vec{L}\ ; \quad (+) = (-)\times(-1)^L \quad \rightarrow \quad L = 1,\ S = 1$$

Transition 2: **$0^+ \rightarrow 0^+$** This is an **allowed Fermi decay**:

$$\vec{0} = \vec{0} + \vec{S} + \vec{L}\ ; \quad (+) = (+)\times(-1)^L \quad \rightarrow \quad L = 0,\ S = 0$$

Transition 3: **$0^+ \rightarrow 1^+$** This is an **allowed Gamow-Teller decay**

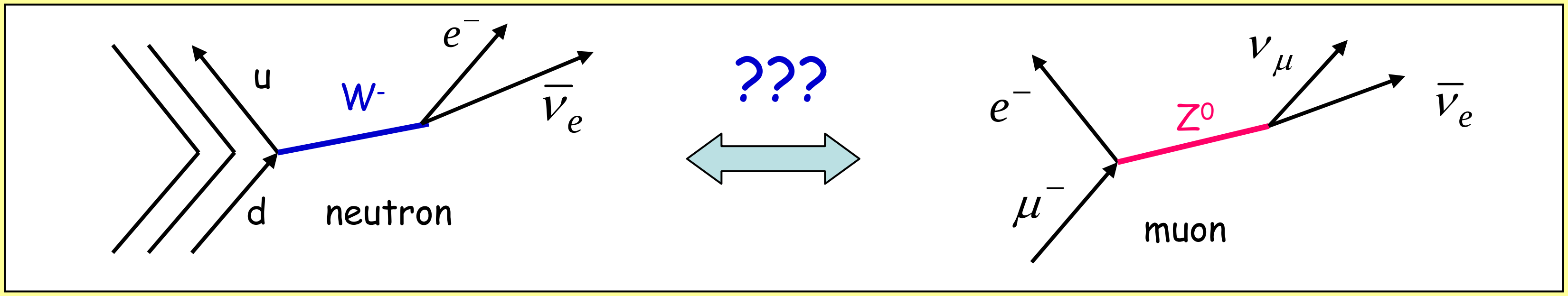
$$\vec{0} = \vec{1} + \vec{S} + \vec{L}\ ; \quad (+) = (+)\times(-1)^L \quad \rightarrow \quad L = 0,\ S = 1$$

We have, so far two coupling constants for (nuclear) beta decay: G_V (S=0 decays) and G_A (S=1). These set the overall scale of the interaction, with G_V determined from the transition rates for "superallowed" 0+ → 0+ nuclear decays, and G_A from Gamow-Teller decays (0+ → 1+ and vice versa).

Other related processes:

1. **muon decay:** $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \quad or \quad \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$

- a "purely leptonic" weak decay -- no quarks before or after!
- no change of electric charge; must be "mediated" by the neutral Z^0 boson
- no "Fermi function" needed, since no Coulomb effects in the final state.
- analogous to neutron decay, so we can try the same formalism, assuming weak interactions for quarks and leptons are the same



Muon lifetime implications:

measured lifetime: τ = 2.19703 ± 0.00004 μs

$$\mu^{\pm} \to e^{\pm} + \nu_e / \overline{\nu}_e \; + \overline{\nu}_\mu / \nu_\mu$$

theoretical prediction:

$$\tau = \frac{192 \; \pi^3 \; \hbar^7}{G^2 \; m_\mu^5 \; c^4}$$

(our prediction, integrated over phase space for the two neutrino types!)

Muon decay gives a weak coupling constant G that is about 2.5% larger than in nuclear beta decays....

or alternatively, the coupling constant for the $d \to u$ quark weak transition is about 2.5% smaller than that for the μ → e lepton weak transition.

2. Pion decay: $\pi^- \to \pi^0 + e^- + \overline{\nu}_e$

Another d → u quark transition; rate is consistent with the same coupling constants as nuclear beta decay

3. K meson decay $K^- \to \pi^0 + e^- + \overline{\nu}_e \qquad (\overline{u}s \to \overline{u}u + e^- + \overline{\nu}_e)$

This is an s → u quark transition; rate is much smaller than the equivalent d → u rate; coupling constants are reduced to about 20% of nuclear beta decay values ....

Weak interactions of quarks:

- There are hundreds of examples of weak decays in nuclear and particle physics.
- Purely leptonic rates are all consistent with a single weak coupling constant G
- Hadronic rates, involving quark transitions, occur at a comparable scale but with consistent differences that depend on the type of quarks involved.
- A simple pattern emerges if we assume that the quarks that participate in weak interactions are linear combinations of the strong interaction eigenstates, represented by a unitary matrix called the CKM (Cabbibo-Kobayashi-Maskawa) matrix:

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \times \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

weak eigenstates

Unitary matrix, like a rotation matrix – preserves “length”

strong eigenstates

$$d' = V_{ud}\, d + V_{us}\, s + V_{ub}\, b, \quad etc...$$

How does this work?

- Instead of a $d \rightarrow u$ transition in neutron beta decay, only the contribution from the weak eigenstate d' plays a role, and the weak coupling constant is effectively reduced by a factor $V_{ud} = 0.974$.
- Similarly, instead of an $s \rightarrow u$ transition in kaon decay, we have an $s' \rightarrow u$ transition, effectively reducing the weak coupling constant by a factor $V_{us} = 0.220$.
- Studies of a large number of particle decays and beta transitions have effectively "mapped out" the CKM matrix as follows: (Particle Data Group , 2004)

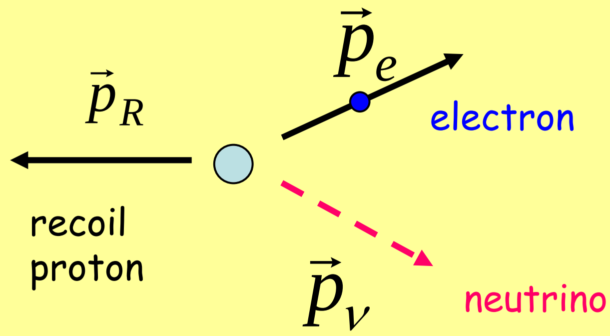
$$\left| \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \right| = \begin{bmatrix} 0.974 & 0.220 & 0.004 \\ 0.224 & 0.996 & 0.041 \\ 0.009 & 0.041 & 0.999 \end{bmatrix}$$

0.9739 to 0.9751	0.221 to 0.227	0.0029 to 0.0045
0.221 to 0.227	0.9730 to 0.9744	0.039 to 0.044
0.0048 to 0.014	0.037 to 0.043	0.9990 to 0.9992

2σ limits

- Diagonal terms dominate the CKM matrix
- All "large" terms are real; small imaginary component in lower right 2x2 submatrix allows for time reversal, or alternatively "CP violation" -- a hot research topic!

Finally, what can we learn from neutron decay?

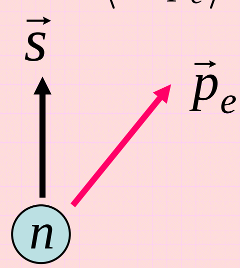


measuring spin-momentum correlations for the decay of polarized neutrons yields additional information (neutron spin: $\vec{s}$)
→ correlation coefficients: a , A, B:

$$A = -2\frac{-G_A G_V + G_A^2}{G_V^2 + 3G_A^2}, \qquad \tau = \frac{\text{constant}}{G_V^2 + 3G_A^2}$$

$$\lambda_{if} \propto p_e E_e \left(Q - E_e\right)^2 \left[1 + a\frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} + \langle \vec{s} \rangle \cdot \left(A\frac{\vec{p}_e}{E_e} + B\frac{\vec{p}_\nu}{E_\nu} \right)\right] dE_e d\Omega_e d\Omega_\nu$$

β asymmetry A, the correlation $\langle \vec{s} \bullet \vec{p}_e \rangle$



$$\vec{n} \rightarrow p + e^- + \bar{\nu}_e$$

"little a" and "B" are hard to measure because one cannot determine the neutrino momentum directly.

The best additional measurement is the "big A" coefficient, which gives an independent constraint from the neutron lifetime, but one has to control and measure the neutron spin direction and measure the electron momentum / energy very precisely…

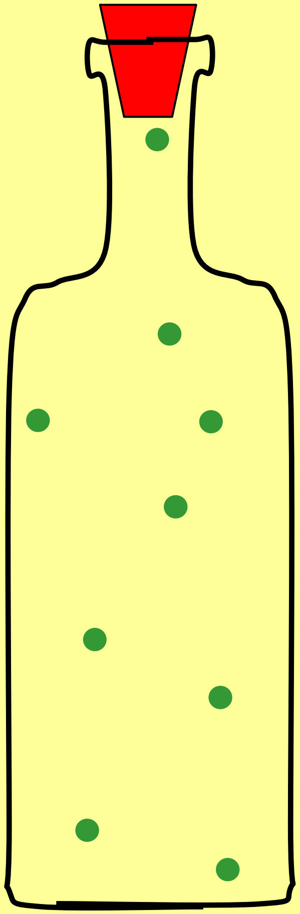
new experiment with ultra cold neutrons:

http://www.krl.caltech.edu/ucn/

What are Ultra Cold Neutrons (UCN) ?

- UCN are neutrons that are moving so slowly that they are totally reflected from a variety of materials.

- They can be confined in material bottles for long periods of time.

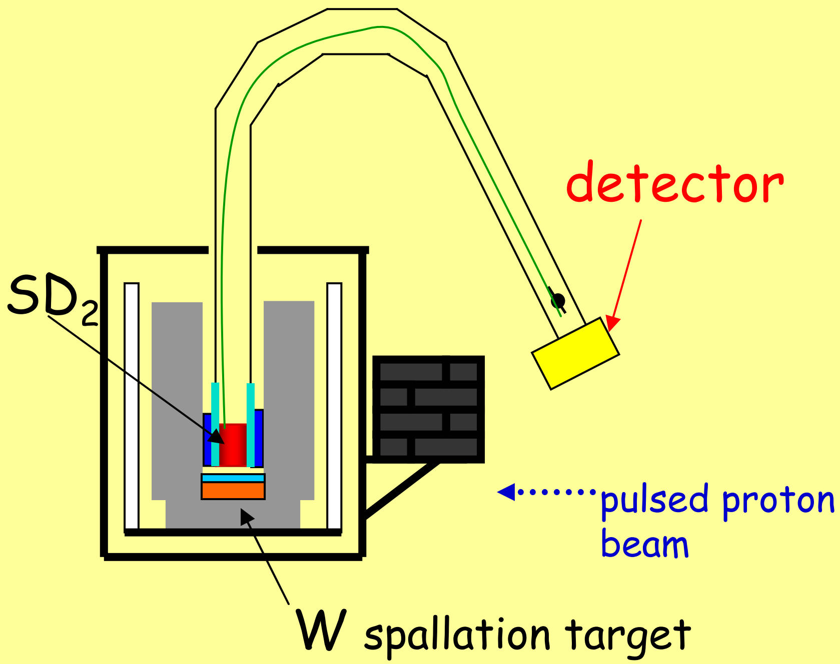


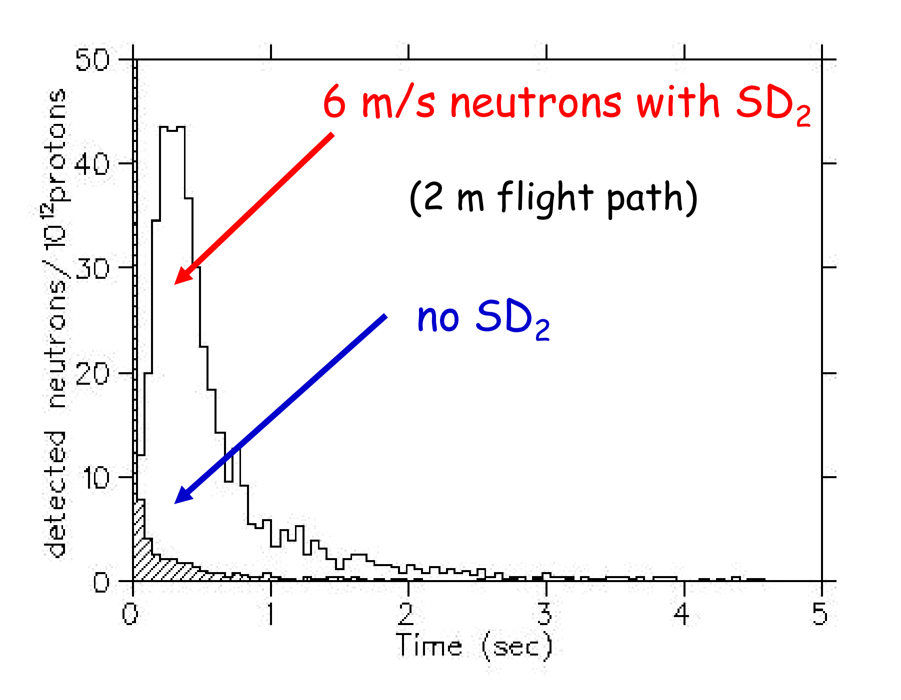
- Typical parameters:
  - velocity < 8 m/s
  - temperature < 4 mK
  - kinetic energy < 300 neV

- Interactions:
  - gravity: V=mgh
  - weak interaction (allows UCN to decay)
  - magnetic fields: $V=-\mu \bullet B$
    (100 % polarization by passing through a magnet !)
  - strong interaction

slides courtesy Prof. J. Martin, U. Wpg.

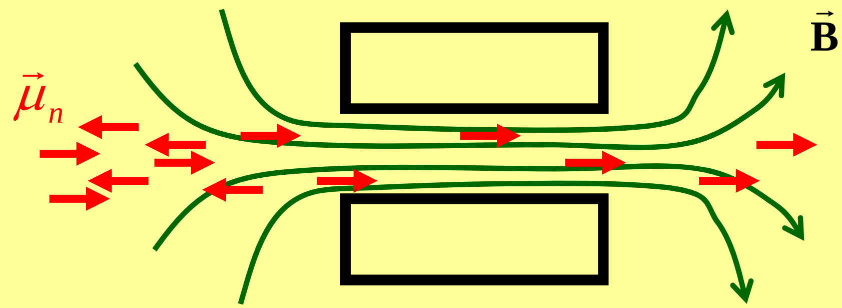
First UCN generation at Los Alamos:





UCNA Advantages: Polarization and Background

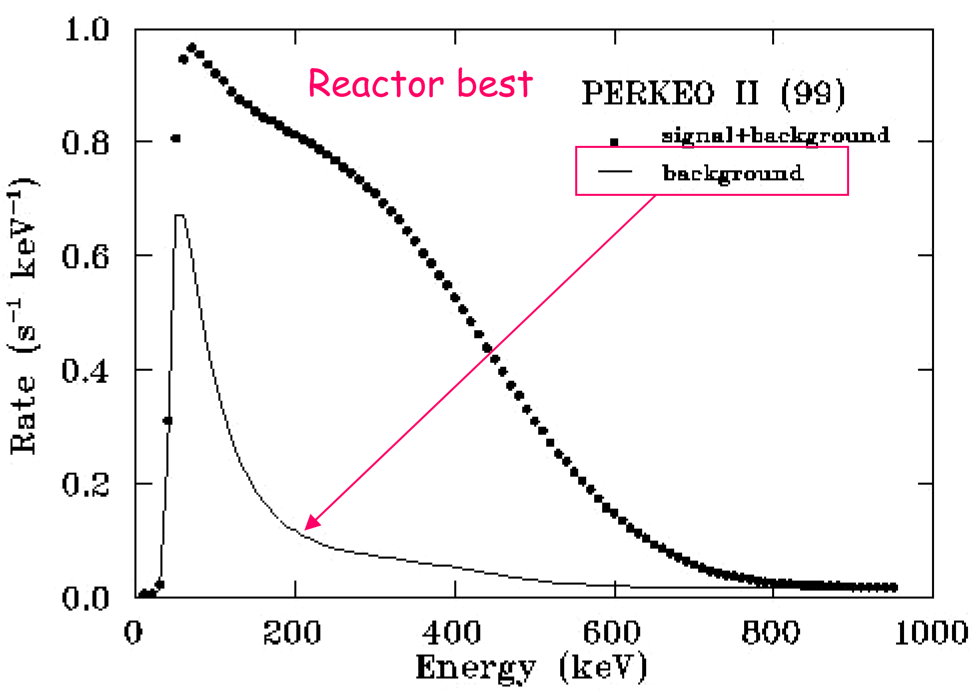
Ultra cold neutrons with the wrong spin direction can't make it through a large magnetic field!

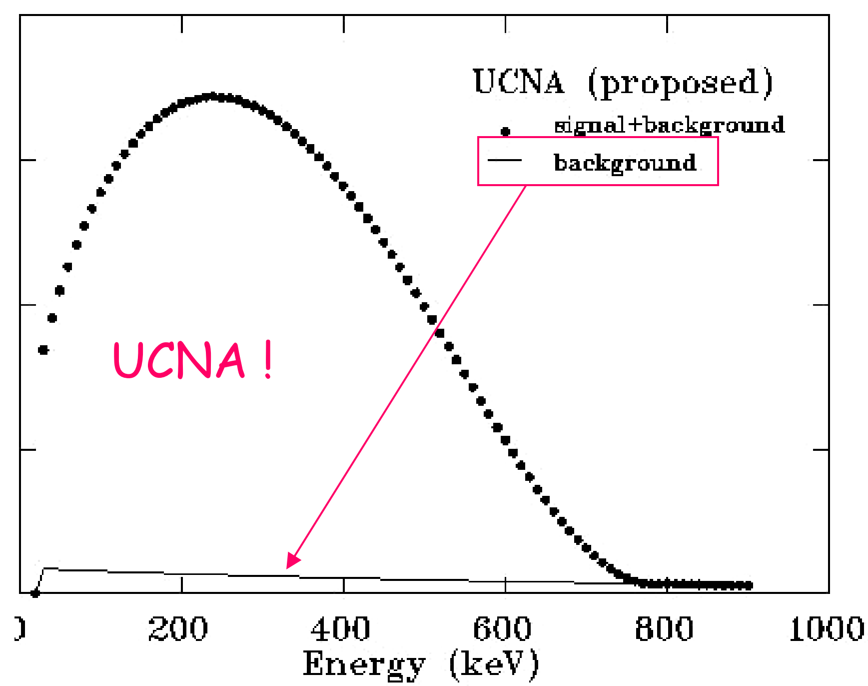


B > 6 T
→ P = 100%

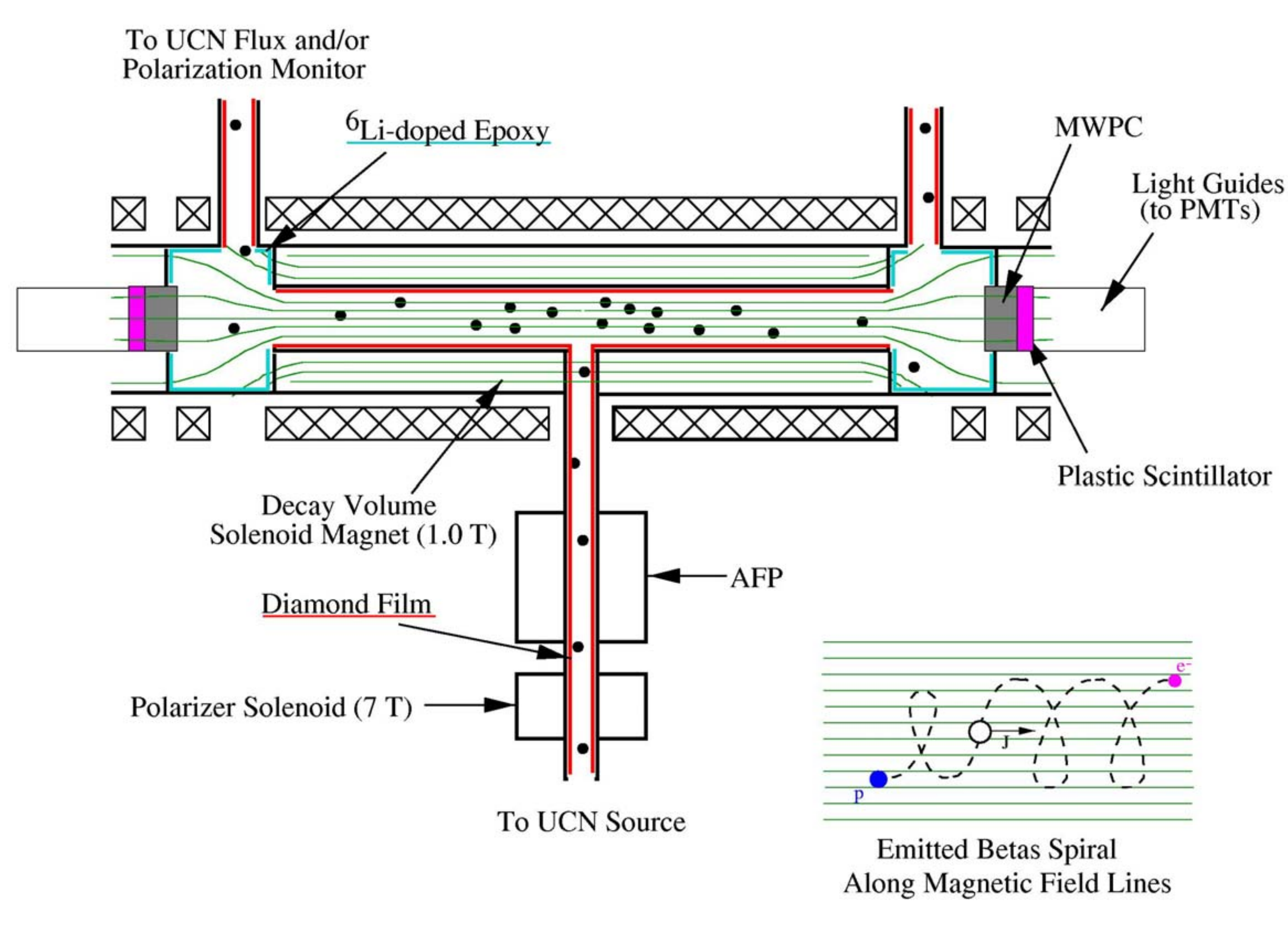
limitation – magnetic scattering from walls, etc.

Background reduction via pulsed source:





 recall: correlation $< \vec{s} \bullet \vec{p}_e > / E_e$



Issues: electron backscattering from detector surface (similar issue in lifetime expt.)
neutron depolarization by scattering from the walls (<~ 0.1 %)

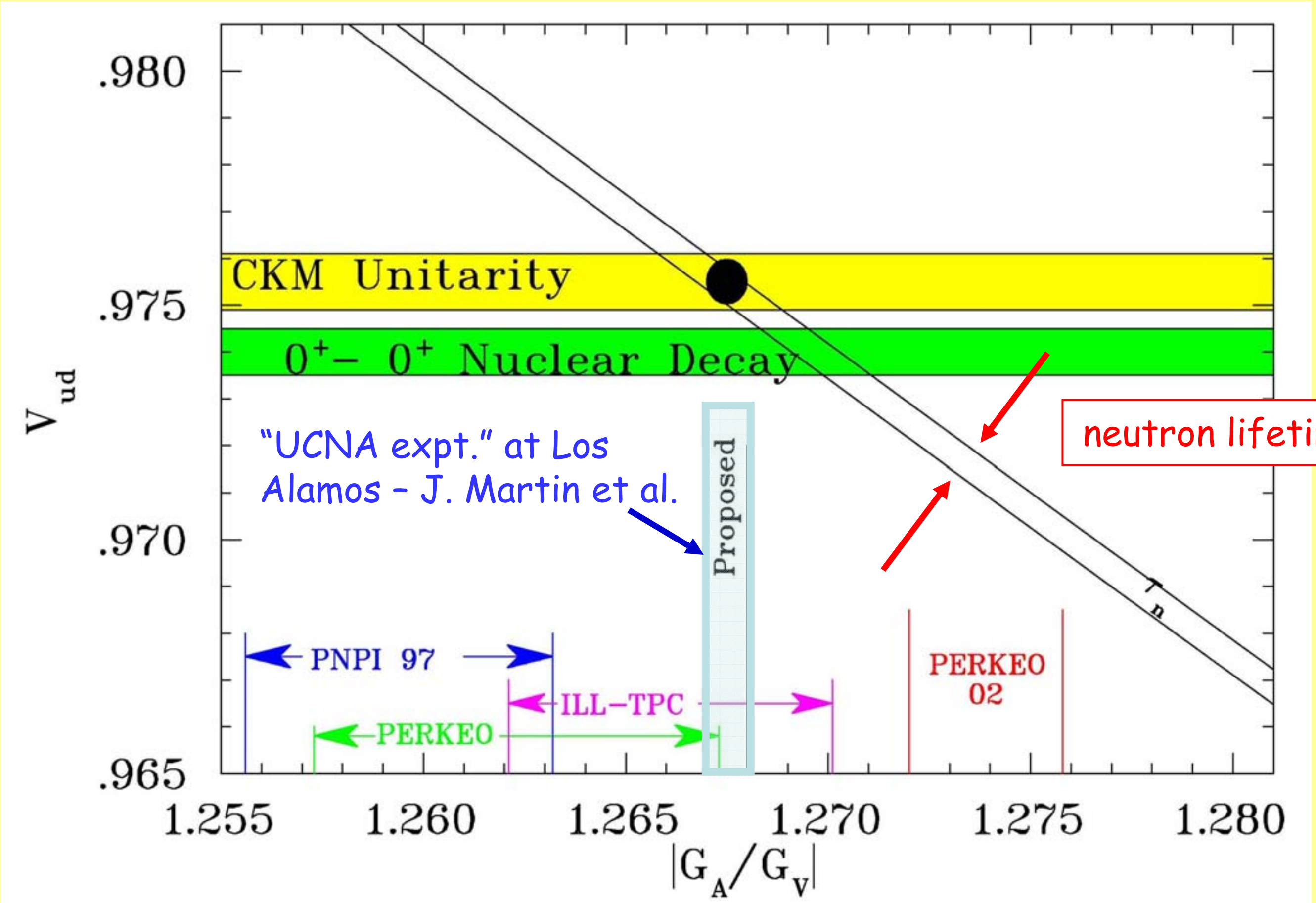
CKM Unitarity test: are there more than three generations of quarks?

$$V_{ij}^{-1} = V_{ij}^{*} \rightarrow V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1 \quad ?$$

(the best-tested row of V_{ij})

world data, 2004: $\boxed{V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.9967 \pm 0.0014}$

2σ discrepancy: contentious issue...



horizontal scale – measurements of "A" in neutron decay

an additional generation of quarks is not entirely ruled out by the CKM unitarity test at the present time!